

Trig Equations for Modelling

1. The distance that water reaches below the top of a rock cliff on an island is measured. Over a period of time, it is found that the maximum distance from the top of the cliff to the water is 12.5 metres and the minimum is 8.7 metres. This cycle repeats after 12 hours.

(a) Give the formula that models the distance of the water (d metres) from the top of the rock cliff, in terms of t hours, starting from low tide.

Hint: drawing a diagram may help.

(b) What is the mean distance of the water below the rock cliff top?

(c) Give a general solution for when the distance is exactly 10 metres.

(d) To dive into the water safely, the distance needs to be less than 10 metres. How many hours a day is it safe to dive?

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2. There is a tourist paddle steamer that goes around the Waitemata harbour. The first blade of the paddle wheel is in upright position (pointing upwards) initially. As the paddle wheel turns, the height of the tip of the first paddle blade ranges from 2.4 m above the deck down to 0.6 m below the deck. The paddle wheel takes 1.8 seconds to do a complete turn when the paddle steamer is at cruising speed.

- (a) Write a trigonometric equation that will model the height of the tip of the first paddle blade as it rotates.

where h = the height in metres

and t = time in seconds.

and the angle is measured in radians.

- (b) Calculate for how long the tip of the first paddle blade is under the deck each time the paddle wheel rotates.

3. On Norfolk Island the number of hours of daylight in each day of the year varies from 10 hours to 14 hours over the 365 days in the calendar year.

The longest day of the year occurs 10 days before the beginning of the calendar year.

- (a) Write a trigonometric equation that will model the number of hours of daylight in each day of the year:

where h is the hours of daylight

and d is the number of days after midnight from the start of the year.

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- (b) How many hours of daylight are there on day one of the year?

- (c) Their prices are highest when the hours of daylight per day are 13 hours or more. This is called “high season”.

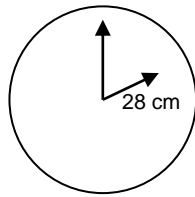
How many days after the beginning of the year does “high season” end?

- (d) Norfolk Island Tourism tries to encourage visitors to the the island during the winter months. They offer special deals for those visiting the island during the time of year when there are less than 11 hours of daylight.

For how many days of the year are the special deals available to visitors?

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4. There is a large clock at the front of the lounge on the paddle steamer. The hour hand is 28 cm long and takes 12 hours to rotate once. At 2 o'clock the tip of the hour hand is 195 cm above the floor.



Floor

- (a) Write a trigonometric equation that will model the height of the tip of the hour hand above the floor.

where h = the height above the floor in centimetres

and t = time since 12 o'clock in hours.

and the angle is measured in radians.

- (b) At what two times is the tip of the hour hand 175 cm above the floor?

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SOLUTIONS:

1. (a) $d = 1.9 \cos \frac{\pi t}{6} + 10.6$

(b) 10.6 metres

(c) $t = 12n \pm 3.61$ or ± 3 hr 37 min

(d) From 3.61 min to 8.39, then 15.61 to 20.39

$$= 2(4.78) = 9.56 \text{ hours} = 9 \text{ hr } 34 \text{ min}$$

2 a) $h = 0.9 + 0.5 \cos \frac{10\pi}{9} t$

b) $t_1 = 0.6343$

$$t_2 = 1.1657$$

Time under the deck = 0.531 sec

3a) $h = 12 + 2 \cos \left[\frac{2\pi}{365} (d + 10) \right]$

b) 13.96 hours of daylight ($d = 1$)

c) 50.8 = 51 days after the beginning of the year

d) 121.67 i.e. 122 days with less than 11 hours of daylight.

4 a) $h = 181 + 28 \cos \frac{\pi}{6} t$

b) $t_1 = 3.41245$

i.e. 3:24:45

$$t_2 = 8.587545$$

i.e. 8:35:15